

Exercice sur la régression linéaire simple – avec la calculatrice.

Un statisticien, *Frank Anscombe*, a construit cet ensemble de données pour illustrer un aspect de la régression linéaire simple. Entrer les données suivantes dans votre calculatrice.

X_1	Y_1	Y_2	Y_3	X_4	Y_4
10	8,04	9,14	7,46	8	6,58
8	6,95	8,14	6,77	8	5,76
13	7,58	8,74	12,74	8	7,71
9	8,81	8,77	7,11	8	8,84
11	8,33	9,26	7,81	8	8,47
14	9,96	8,1	8,84	8	7,04
6	7,24	6,13	6,08	8	5,25
4	4,26	3,1	5,39	19	12,5
12	10,84	9,13	8,15	8	5,56
7	4,82	7,26	6,42	8	7,91
5	5,68	4,74	5,73	8	6,89

- a) Compléter à l'aide de la calculatrice le tableau suivant. Qu'est-ce que ces modèles ont en commun ? si on se fie à ces résultats, peut-on conclure que ces quatre séries de données sont similaires?

Modèle	b_0	b_1	R^2	r
$Y_1 = b_0 + b_1 X_1$				
$Y_2 = b_0 + b_1 X_1$				
$Y_3 = b_0 + b_1 X_1$				
$Y_4 = b_0 + b_1 X_4$				

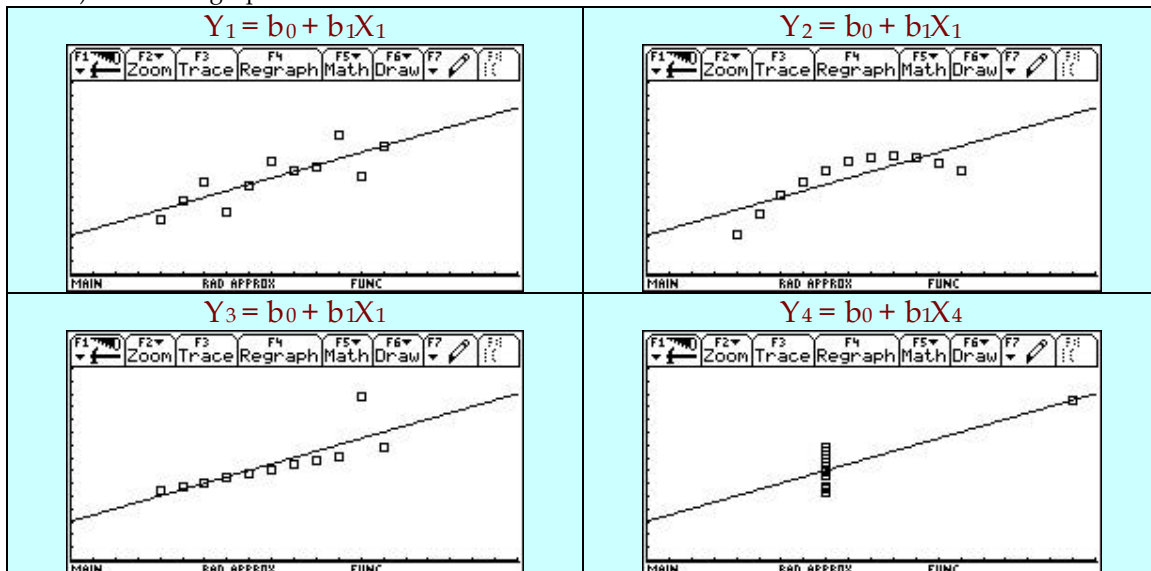
- b) Obtenir les quatre nuages de points avec leurs droites de régressions.
Expliquer ce que *Frank Anscombe* cherchait à illustrer à l'aide de ces données.
- c) Tester chacun de ces modèles avec la T.I. au seuil $\alpha = 5\%$.
- d) Pour chacun des modèles, donner un intervalle de confiance à 95% pour estimer la valeur moyenne de Y lorsque $X = 12$ et fournir la marge d'erreur (E).

Solution

a) Modèle	b_0	b_1	R^2	r
$Y_1 = b_0 + b_1X_1$	3,00009	0,500091	0,666542	0,816421
$Y_2 = b_0 + b_1X_1$	3,00091	0,500000	0,666242	0,816237
$Y_3 = b_0 + b_1X_1$	3,00245	0,499727	0,666324	0,816287
$Y_4 = b_0 + b_1X_4$	3,00173	0,499909	0,666707	0,816521

A partir de ces données on serait porté à penser qu'on a des données similaires

b) Voici les graphes :



Donc, il ne faut pas sauter trop vite aux conclusions ... on a quatre situations fort différentes !

c) Test : $H_0 : r = 0$ (modèle non significatif) versus $H_1 : r \neq 0$ (modèle significatif)

Modèle	t	P-value	Rejet de H_0
$Y_1 = b_0 + b_1X_1$	4,24146	0,00217	oui
$Y_2 = b_0 + b_1X_1$	4,23859	0,002179	oui
$Y_3 = b_0 + b_1X_1$	4,23937	0,002176	oui
$Y_4 = b_0 + b_1X_4$	4,24303	0,002165	oui

d) Intervalles de confiance pour la moyenne des Y quand $X = 12$ (confidence limits)

Modèle	$\hat{Y}_p$	Li	Ls	E
$Y_1 = b_0 + b_1X_1$	9,00118	7,839	10,16	1,16261
$Y_2 = b_0 + b_1X_1$	9,00091	7,838	10,16	1,16318
$Y_3 = b_0 + b_1X_1$	8,99918	7,837	10,16	1,16233
$Y_4 = b_0 + b_1X_4$	9,00064	7,839	10,16	1,16176

Voir ci-dessous pour les détails.

Saisie des données

F1▼ Tools	F2▼ Plots	F3▼ List	F4▼ Calc	F5▼ Distr	F6▼ Tests	F7▼ Ints	
lex1	ley1	ley2	ley3	lex4	ley4		
10.	8.04	9.14	7.46	8.	6.58		
8.	6.95	8.14	6.77	8.	5.76		
13.	7.58	8.74	12.74	8.	7.71		
9.	8.81	8.77	7.11	8.	8.84		
11.	8.33	9.26	7.81	8.	8.47		
14.	9.96	8.1	8.84	8.	7.04		
6.	7.24	6.13	6.08	8.	5.25		
4.	4.26	3.1	5.39	19.	12.5		
lex1={10., 8., 13., 9., 11., 14., 6., 4.}							
MAIN		RAD APPRX		FUNC 1/6			

F1▼ Tools	F2▼ Plots	F3▼ List	F4▼ Calc	F5▼ Distr	F6▼ Tests	F7▼ Ints
lex1	ley1	ley2	ley3	lex4	ley4	
10.	8.04	9.14	7.46	8.	6.58	
8.	6.95	8.14	6.77	8.	5.76	
13.	7.58	8.74	12.74	8.	7.71	
9.	8.81	8.77	7.11	8.	8.84	
11.	8.33	9.26	7.81	8.	8.47	
14.	9.96	8.1	8.84	8.	7.04	
6.	7.24	6.13	6.08	8.	5.25	
4.	4.26	3.1	5.39	19.	12.5	
lex1={10., 8., 13., 9., 11., 14., 6..}						
MAIN			RAD APPROR		FUNC 1 / 6	

- o F4 (Calc)
- o 3 (Regressions)
- o 1 (LinReg(a+bx))

- o F4 (Calc)
- o 3 (Regressions)
- o 1 (LinReg(a+bx))

LinReg

Y1=aX+bX+c...

X List: L1

Y List: L1

Store RegEqn to: Y1(X)→

Freq:

Category List:

Include Categories: ☒

ENTER=OK ESC=CANCEL

Y1=aX+bX+c...

TYPE<ENTER>OK and ESC=CANCEL

TI-84 Plus calculator screen showing a linear regression calculation. The screen displays the equation $y=a+bx$ and the following statistics:

- $a = 3.00009$
- $b = -.500091$
- $r^2 = .666542$
- $r = .816421$

The "Enter=OK" button is highlighted. The data list L1 is shown at the bottom: $L1 = \{10, 8, 13, 9, 14, 6, 4\}$.

- o ♦ Y=
- o choisir « Plot 1 : »
- o F3 (Edit)

- ```
0 ◆ Y=
0 choisir « Plot 1 : »
0 F3 (Edit)
```

Define Plot 1

Plot Type..... Scatter+  
 Mark..... Box+  
 x..... lex1  
 y..... lev1  
 Hist. Overlay Width..... 1  
 Use Freq and Categories? NO+  
 Frequency.....  
 Category.....  
 Include Categories..... C  
 Enter=SAVE ESC=CANCEL

Avec ♦ Window



The screenshot shows the TI-84 Plus CSE calculator interface. At the top, the 'F1' and 'F2' function keys are visible, with 'F2' labeled 'ZOOM'. Below the function keys, the following settings are displayed:

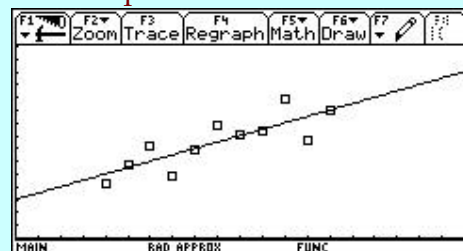
- xmin=0.
- xmax=20.
- xsc1=1.
- ymin=0.
- ymax=15.
- ysc1=1.
- xres=2.

At the bottom of the screen, the 'MAIN' menu is visible, with 'END APPEND' and 'FUNC' options.

The screenshot shows the TI-84 Plus C calculator screen with the ZOOM menu open. The menu options are: xmin=0, xmax=20, xscl=1, ymin=0, ymax=15, yscl=1, and xres=2. The cursor is positioned on the ZOOM option.

## Avec ♦ Graph

The image shows a TI-84 Plus calculator screen in the 'Graph' mode. The top status bar displays 'F1', 'F2', 'F3', 'F4', 'F5', 'F6', 'F7', and 'F8' with corresponding icons. The main display area shows a scatter plot with approximately 12 data points (small squares) and a solid line of best fit. The x-axis is labeled 'MAIN' and the y-axis is labeled 'RAD APPR' and 'FINC'. The bottom status bar shows 'MAIN', 'RAD APPR', and 'FINC'.



Pour tester le modèle

- o **F6** ( Tests )
- o A :LinRegTTest...

| F1                       | F2                                | F3           | F4         | F5    | F6    | F7   |
|--------------------------|-----------------------------------|--------------|------------|-------|-------|------|
| Tools                    | Plots                             | List         | Calc       | Distr | Tests | Ints |
| Linear Regression T Test |                                   |              |            |       |       |      |
| 1ex1                     | X List:                           | 1ex1         |            | 421   |       | esid |
| 8.04                     | Y List:                           | 1ex1         |            | .241  |       |      |
| 6.95                     | Freq:                             | 1            |            | .709  |       |      |
| 7.58                     | Alternate Hyp:                    | $\mu \neq 0$ |            | .839  |       |      |
| 8.81                     | Store RegEqn to:                  | none         |            | .469  |       |      |
| 8.33                     | Results:                          | Calculate    |            | .039  |       |      |
| 9.96                     |                                   |              |            | .751  |       |      |
| 7.24                     | Enter=OK                          |              | ESC=CANCEL |       |       |      |
| 4.26                     | resid={-.42100000000002, -1.24... |              |            |       |       |      |
| MAIN RAD APPROX FUNC 2/2 |                                   |              |            |       |       |      |

| F1                                | F2           | F3   | F4   | F5      | F6    | F7    |
|-----------------------------------|--------------|------|------|---------|-------|-------|
| Tools                             | Plots        | List | Calc | Distr   | Tests | Ints  |
| Linear Regression T Test          |              |      |      |         |       |       |
| 1ex1                              | y=a+bx       |      |      |         |       | esid  |
| 8.04                              | $\mu \neq 0$ |      |      | 4.24146 |       | .421  |
| 6.95                              | t            |      |      | -.00217 |       | 1.241 |
| 7.58                              | P Value      |      |      | .00217  |       | .709  |
| 8.81                              | df           |      |      | 9.      |       | .839  |
| 8.33                              | a            |      |      | 3.00009 |       | .469  |
| 9.96                              | b            |      |      | 5.00091 |       | .039  |
| 7.24                              | s            |      |      | 1.2366  |       | 1.751 |
| 4.26                              | SE Slope     |      |      | .117906 |       |       |
|                                   | $r^2$        |      |      | .666542 |       |       |
| Enter=OK                          |              |      |      |         |       |       |
| resid={-.42100000000002, -1.24... |              |      |      |         |       |       |
| MAIN RAD APPROX FUNC 2/2          |              |      |      |         |       |       |

Pour l'intervalle de confiance

- o **F7** ( Ints )
- o 7 :LinRegTInt...

| F1                           | F2                                | F3       | F4         | F5    | F6    | F7    |
|------------------------------|-----------------------------------|----------|------------|-------|-------|-------|
| Tools                        | Plots                             | List     | Calc       | Distr | Tests | Ints  |
| Linear Regression T Interval |                                   |          |            |       |       |       |
| 1ex1                         | X List:                           | 1ex1     |            |       |       | esid  |
| 8.04                         | Y List:                           | 1ex1     |            | .0508 |       | .421  |
| 6.95                         | Freq:                             | 1        |            | .921  |       | 1.241 |
| 7.58                         | Store RegEqn to:                  | none     |            | .3091 |       | .709  |
| 8.81                         | Interval:                         | Response |            | .1711 |       | .839  |
| 8.33                         | X Value:                          | 12       |            | .2394 |       | .469  |
| 9.96                         |                                   |          |            | .7405 |       | .039  |
| 7.24                         | Enter=OK                          |          | ESC=CANCEL |       |       |       |
| 4.26                         | resid={.039000000000001, -.050... |          |            |       |       |       |
| MAIN RAD APPROX FUNC 2/2     |                                   |          |            |       |       |       |

| F1                                | F2       | F3   | F4   | F5             | F6    | F7    |  |
|-----------------------------------|----------|------|------|----------------|-------|-------|--|
| Tools                             | Plots    | List | Calc | Distr          | Tests | Ints  |  |
| Lin Reg T Interval - Response     |          |      |      |                |       |       |  |
| 1ex1                              | y=a+bx   |      |      |                |       | esid  |  |
| 8.04                              | y_hat    |      |      | 9.00118        |       | .421  |  |
| 6.95                              | df       |      |      | 9.             |       | 1.241 |  |
| 7.58                              | C Int    |      |      | (7.839, 10.16) |       | .709  |  |
| 8.81                              | ME       |      |      | 1.16261        |       | .839  |  |
| 8.33                              | SE       |      |      | .513938        |       | .469  |  |
| 9.96                              | Pred Int |      |      | (5.972, 12.03) |       | .039  |  |
| 7.24                              | ME       |      |      | 3.02937        |       | 1.751 |  |
| 4.26                              | Enter=OK |      |      |                |       |       |  |
| resid={.039000000000001, -.050... |          |      |      |                |       |       |  |
| MAIN RAD APPROX FUNC 2/2          |          |      |      |                |       |       |  |

Répéter ces étapes pour obtenir les régressions de ...

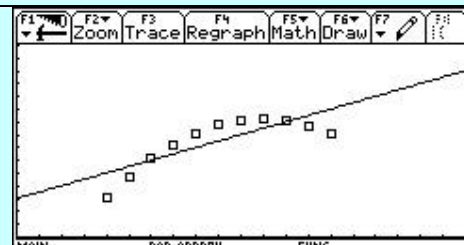
- o Y2 sur X1
- o Y3 sur X1
- o Y4 sur X4

**Y2 sur X1**

| F1               | F2                                     | F3     | F4         | F5    | F6    | F7   |
|------------------|----------------------------------------|--------|------------|-------|-------|------|
| Tools            | Plots                                  | List   | Calc       | Distr | Tests | Ints |
| LinReg3(a+bx)... |                                        |        |            |       |       |      |
| 1ex1             | X List:                                | 1ex1   |            |       |       |      |
| 10.              | Y List:                                | 1ex2   |            |       |       |      |
| 8.               | Store RegEqn to:                       | y2(x)→ |            |       |       |      |
| 13.              | Freq:                                  | 1      |            |       |       |      |
| 9.               | Category List:                         |        |            |       |       |      |
| 11.              | Include Categories:                    |        |            |       |       |      |
| 14.              | Enter=OK                               |        | ESC=CANCEL |       |       |      |
| 6.               | 1ex1={10., 8., 13., 9., 11., 14., 6... |        |            |       |       |      |
| 4.               | MAIN RAD APPROX FUNC 1/2               |        |            |       |       |      |

| F1               | F2                                     | F3   | F4   | F5      | F6    | F7   |  |
|------------------|----------------------------------------|------|------|---------|-------|------|--|
| Tools            | Plots                                  | List | Calc | Distr   | Tests | Ints |  |
| LinReg3(a+bx)... |                                        |      |      |         |       |      |  |
| 1ex1             | y=a+bx                                 |      |      |         |       | ey4  |  |
| 10.              | a                                      |      |      | 3.00091 |       | .58  |  |
| 8.               | b                                      |      |      | .5      |       | .76  |  |
| 13.              | $r^2$                                  |      |      | .666242 |       | .71  |  |
| 9.               | r                                      |      |      | .816237 |       | .84  |  |
| 11.              | Enter=OK                               |      |      |         |       |      |  |
| 14.              | 1ex1={10., 8., 13., 9., 11., 14., 6... |      |      |         |       |      |  |
| 6.               | MAIN RAD APPROX FUNC 1/2               |      |      |         |       |      |  |

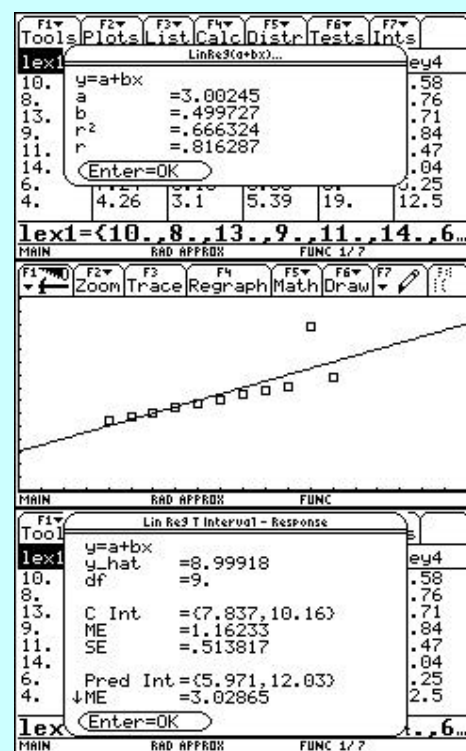
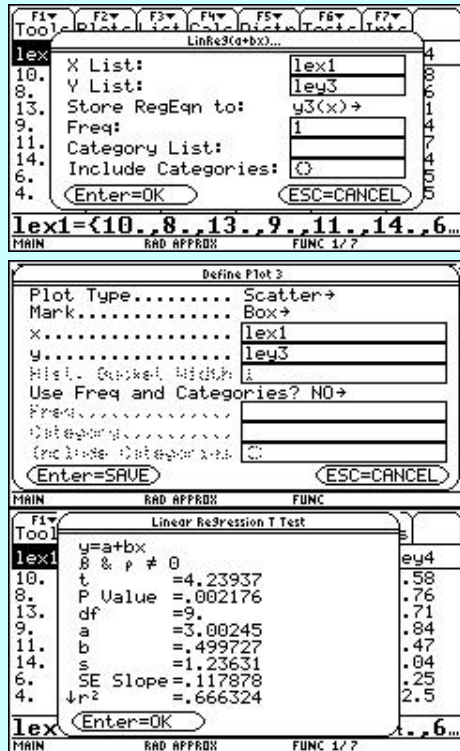
| F1                           | F2    | F3         | F4   | F5    | F6    | F7   |
|------------------------------|-------|------------|------|-------|-------|------|
| Tools                        | Plots | List       | Calc | Distr | Tests | Ints |
| Define Plot 2                |       |            |      |       |       |      |
| Plot Type..... Scatter→      |       |            |      |       |       |      |
| Mark..... Box→               |       |            |      |       |       |      |
| X..... 1ex1                  |       |            |      |       |       |      |
| Y..... 1ex2                  |       |            |      |       |       |      |
| Hist. Bucket Width: 1        |       |            |      |       |       |      |
| Use Freq and Categories? NO→ |       |            |      |       |       |      |
| Freq.....                    |       |            |      |       |       |      |
| Category.....                |       |            |      |       |       |      |
| Include Categories: C        |       |            |      |       |       |      |
| Enter=SAVE                   |       | ESC=CANCEL |      |       |       |      |
| MAIN RAD APPROX FUNC         |       |            |      |       |       |      |



| F1                                     | F2           | F3   | F4   | F5       | F6    | F7   |
|----------------------------------------|--------------|------|------|----------|-------|------|
| Tools                                  | Plots        | List | Calc | Distr    | Tests | Ints |
| Linear Regression T Test               |              |      |      |          |       |      |
| 1ex1                                   | y=a+bx       |      |      |          |       | ey4  |
| 10.                                    | $\mu \neq 0$ |      |      | 4.23859  |       | .58  |
| 8.                                     | t            |      |      | -.002179 |       | .76  |
| 13.                                    | P Value      |      |      | .002179  |       | .71  |
| 9.                                     | df           |      |      | 9.       |       | .84  |
| 11.                                    | a            |      |      | 3.00091  |       | .47  |
| 14.                                    | b            |      |      | .5       |       | .04  |
| 6.                                     | s            |      |      | 1.23721  |       | .25  |
| 4.                                     | SE Slope     |      |      | .117964  |       | 2.5  |
|                                        | $r^2$        |      |      | .666242  |       |      |
| Enter=OK                               |              |      |      |          |       |      |
| 1ex1={10., 8., 13., 9., 11., 14., 6... |              |      |      |          |       |      |
| MAIN RAD APPROX FUNC 1/2               |              |      |      |          |       |      |

| F1                            | F2                                 | F3   | F4   | F5             | F6    | F7     |  |
|-------------------------------|------------------------------------|------|------|----------------|-------|--------|--|
| Tools                         | Plots                              | List | Calc | Distr          | Tests | Ints   |  |
| Lin Reg T Interval - Response |                                    |      |      |                |       |        |  |
| 1ex1                          | y=a+bx                             |      |      |                |       | esid   |  |
| 10.                           | y_hat                              |      |      | 9.00091        |       | .1391  |  |
| 8.04                          | df                                 |      |      | 9.             |       | .1391  |  |
| 6.95                          | C Int                              |      |      | (7.838, 10.16) |       | .7609  |  |
| 7.58                          | ME                                 |      |      | 1.16318        |       | .2691  |  |
| 8.81                          | SE                                 |      |      | .514192        |       | .75909 |  |
| 8.33                          | Pred Int                           |      |      | (5.97, 12.03)  |       | 1.901  |  |
| 9.96                          | ME                                 |      |      | 3.03086        |       | 12909  |  |
| 7.24                          | Enter=OK                           |      |      |                |       |        |  |
| 4.26                          | resid={.139100000000001, -.1391... |      |      |                |       |        |  |
| MAIN RAD APPROX FUNC 2/2      |                                    |      |      |                |       |        |  |

### Y3 sur X1



### Y4 sur X4

